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13. ABSTRACT *(Maximum 200 words)*

This report summarizes work carried out in the specific areas of inverse scattering and optimal control problems in electromagnetics. Major accomplishments involve the modified gradient approach to inverse which was extended to the TE case, inhomogeneous background media, spatially limited frequency diverse data and the invlusion of a resolution enhancing total variation terms. In addition the complete family approach was further developed, high frequency optimal current distributions were found for conformal antennas and additional results on low frequency asymptotics and a-posteriori error estimates in numerical solutions of direct problems were found.

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FINAL TECHNICAL REPORT

TO

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

ON

AFOSR GRANT NO. F49620–94–1–0219

OPTIMIZATION METHODS FOR DIRECT AND INVERSE PROBLEMS IN ELECTROMAGNETICS

Name of Institution:	Center for the Mathematics of Waves Department of Mathematical Sciences University of Delaware Newark, DE 19716

Time Period:	1 July 1994 – 30 June 1996

Principal Investigators:	T. S. Angell R. E. Kleinman

Program Manager:	Arje Nachman

Final Technical Report

AFOSR Grant No. F49620–94–1–0219

1 July 1994 – 30 June 1996

1. Research Objectives:

The research conducted under this grant is a continuation of a long–term research effort devoted to the study of various aspects of electromagnetic radiation and scattering, direct and inverse, previously supported by AFOSR. The general goal of the program continues to be the establishment of a firm mathematical foundation and the development of algorithms based on such a foundation in which boundary and domain parameters are either to be recovered from scattering or radiation data or used as controls to optimize various functionals of the scattered or radiated fields. Such parameters include the shape of the boundary itself, functions defined on the boundary such as impedance, generalized impedance and generalized resistivity, as well as domain parameters such as conductivity and refractive index. The program is focussed on three specific areas of investigation: multicriteria optimization, generalized impedance boundary conditions, and inverse scattering techniques.

In addition we have begun the writing of a monograph devoted to optimization methods in antenna theory which will be devoted, to a large extent, to the systematic exposition of the theory and computational results obtained with the support of several AFOSR grants. This monograph is being written in collaboration with Professor A. Kirsch of the Universität Erlangen–Nürnberg in Germany. It is expected to be completed during his visit to the University of Delaware in early 1997.

The following list encompasses what were the overall research accomplishments of this grant.

- Continuation of work on the monograph on optimization methods in the theory of antennas.
- Extension of work on asymptotic methods which characterize optimal surface currents for conformal antenna problems.
- Expansion of the number of examples treated with multicriteria optimization methods with particular emphasis on comparison of results for optimal antenna synthesis problems which appear in the literature.
- Investigation of the usefulness of multicriteria optimization techniques in the inverse shape identification problem.
- Continuation of work on the theory of higher order boundary conditions with a particular emphasis on the study of the derivation of these conditions for general classes of bounded obstacles.
- Investigation of the use of conductivity, resistivity, and higher order impedances as controls for optimizing radiation and scattering characteristics.
- Continuation of and expansion of the revised complete family approach to shape reconstruction by establishing convergence, testing stability, providing numerical implementation, and extending to time domain.
- Expansion of the modified gradient approach to inverse scattering by seeking better initial choices in the minimization algorithm, deriving a comparable

algorithm for discontinuous fields at the boundary, extending the method to multifrequency spatially limited data.

- Development, together with Rome Laboratory, of a data bank of canonical inverse scattering problems including experimental data sets and reconstructions.

2. Research Accomplishments and Current Status

During the grant period, work was concentrated on two approaches to inverse problems. The modified gradient method developed under the preceding grant was extended in collaboration with P. M. van den Berg, Delft University of Technology, in a number of directions. In its original form the method was applied to reconstructing the (complex) index of refraction of a bounded, inhomogeneous object from scattered field data at a single frequency with multiple incident directions. Dramatic improvement in the resolution of the reconstructed profiles was achieved by including a penalty term involving the total variation of the profile as reported in [3.3].[1] Work is continuing on optimal determination of the penalty parameter. Work began in applying the modified gradient approach in the more complicated situation in which the background media is not homogeneous but consists of two dissimilar homogeneous half spaces. Using scattered field data limited to observation on a line and incident directions limited to one half space we attempted to locate and reconstruct objects imbedded in the other half space. We tried to compensate for the loss of spatial data by using scattering data at multiple frequencies. A number of specialized two dimensional objects were considered and preliminary results were reported in [5.a.2], [5.a.3], [5.a.4]. Two manuscripts describing this work have been completed [3.9] and [3.10]. This work is being performed in collaboration with

[1] Citations refer to papers listed in Section 3 and presentations listed in Section 5.a.

personnel from the Laboratoire des Signaux et Systèmes, CNRS, Ecole Superieure d'Electricité, Gif sur Yvette, France.

A second direction in which the modified gradient approach was extended involved the transverse electric (TE) case. Previous work was limited to the simplest example of $2-D$ electromagnetic inverse scattering in which the electric vector is polarized perpendicular to the cross section of the cylindrical scatterer (the TM case). During this period the algorithm was generalized to the TE case which is more complicated in that the underlying domain integral equation is no longer weakly singular and is now Cauchy singular. Nevertheless the algorithm has been extended and preliminary results were reported in [5.a.6] and [5.a.9]. This work is the subject of Wen Lixin's doctoral dissertation at Delaware. The modified gradient method was also combined with the Newton–Kantorovich approach to produce a hybrid method which performs somewhat differently from each of the component techniques. Preliminary results were reported in [5.a.8] and [5.a.10]. This work is in collaboration with personnel from CNRS–Sophia Antipolis in France and is the dissertation topic of Pierre Lobel at the Université de Nice – Sophia Antipolis. The last extension of the modified gradient method which received attention during this period involved shape and location determination of an impenetrable scatterer. The manuscript describing this previously reported successful shape reconstruction from experimental data in a "blind" test was revised and has appeared in *IEEE–AP–S* [3.6], [3.12], [3.13], [3.14]. These results were also reported in [5.a.1] and [5.a.7].

The second approach to inverse problems which received attention involved the so–called "complete family" method in which the coefficients of a representation of the scattered field as a linear combination of linearly independent solutions of

the underlying field equation together with the coefficients of an expansion of a parametric representation of the scattering surface are found by simultaneously minimizing the discrepancy in matching the far field with measured data and the boundary condition on the surface. A new numerical approach which appears very effective was developed under predecessor grants and the manuscript describing this approach was completed [3.7]. In addition work on the application of this technique to a wave guide problem was initiated in conjunction with collaborators in France. A further application of complete families was employed in [3.8] to illustrate that convergence of the point collocation approach to scattering by a grating does not depend on fulfillment of the so–called Rayleigh hypothesis, contrary to widely held belief. This work was carried out in collaboration with S. Christiansen of the Technical University of Denmark.

Previously we had reported on the problem of determining the current distribution on a surface (antenna) which optimizes radiated power in a sector. For smooth convex surfaces we obtained an explicit asymptotic solution of this problem at high frequencies. The first paper on this work appeared [3.4], results were presented [5.a.5] and a more complete paper including particular examples is under preparation. This work is in collaboration with B. Vainberg, University of North Carolina – Charlotte.

Other activity in related areas under this grant included continued work on the monograph on optimization methods in antenna theory as well as a book on low frequency methods in acoustics, electromagnetics and elasticity. The previously reported derivation of complete low frequency expansions of solutions of two dimensional elliptic problems appeared [3.1]. Finally, increased attention was given

to the problem of error estimation in the numerical solution of integral equations and coupled finite element–boundary integral equations. This work began under an ONR grant however it has great relevance to the numerical aspects of the inverse scattering and antenna optimization problems which form the principal research topics of the present grant. Results are described in [3.2], [3.5] and [3.11].

3. Publications supported under AFOSR Grant; 1 July 1994–30 June 1996.

1. Full Low–Frequency Asymptotic Expansion for Second–Order Elliptic Equations in Two Dimensions, R. E. Kleinman and B. Vainberg, *Math Methods in the Appl. Sci.* **17**, 989–1004, 1994.

2. Asymptotic and A–Posteriori Error Estimates for Boundary element Solutions of Hypersingular Integral Equations, M. Feistauer, G. C. Hsiao and R. E. Kleinman, *SIAM J. Numer. Anal.* **33**, 666–685, 1996.

3. A total variation enhanced modified gradient algorithm for profile reconstruction, P. M. van den Berg and R. E. Kleinman, *Inverse Problems* **11**, 1995.

4. Asymptotic Approximation of Optimal Solutions of an Acoustic Radiation Problem, T. S. Angell, R. E. Kleinman and B. Vainberg, in *Inverse Scattering and Potential Problems in Mathematical Physics*, R. E. Kleinman, R. Kress and E. Martensen, eds. Peter Lang, Frankfurt, 5–16, 1995.

5. Analysis and Numerical Realization of Coupled BEM and FEM for Nonlinear Exterior Problems, M. Feistauer, G. C. Hsiao, R. E. Kleinman and R. Tezaur, in *Inverse Scattering and Potential Problems in Mathematical Physics*, R. E. Kleinman, R. Kress and E. Martensen, eds., Peter Lang, Frankfurt, 47–74, 1995.

6. Blind Shape Reconstruction From Experimental Data, P. M. van den Berg, M. G. Coté and R. E. Kleinman, *IEEE–AP* **43**, 1389–1396, 1995.

7. On a Numerical Method for Inverse Acoustic Scattering, T. S. Angell, R. E. Kleinman and Jinmin Jiang, *Inverse Problems*, to appear.

8. On a Misconception Involving Point Collocation and the Rayleigh Hypothesis, Søren Christiansen and Ralph Kleinman, *IEEE–AP*, to appear.

9. Microwave Imaging–Location and Shape Reconstruction from Multifrequency Scattering Data, K. Belkebir, R. E. Kleinman and Ch. Pichot, *IEEE–MTT*, to appear.

10. A Modified Gradient Approach to Inverse Scattering for Binary Objects in Stratified Media, L. Souriau, B. Duchêne, D. Lesselier and R. E. Kleinman, *Inverse Problems* **12**, 463–481, 1996.

11. Error Control in Numerical Solution of Boundary Integral Equations, G. C. Hsiao and R. E. Kleinman, *ACES*, **11**, 32–36, 1996.

12. Special Session on Image Reconstruction Using Real Data, Robert V. McGahan and Ralph E. Kleinman, *IEEE-AP Magazine* **38(3)**, 39–40, 1996.

13. Conjugate–Gradient Method for Solving Inverse Scattering with Experimental Data, P. Lobel, R. E. Kleinman, Ch. Pichot, L. Blanc–Ferand and M. Barlaud, *IEEE-AP Magazine* **38(3)**, 48–51, 1996.

14. Image Reconstruction From Ipswich Data, Peter van den Berg and Ralph E. Kleinman, *IEEE-AP Magazine* **38(3)**, 56–59, 1996.

4. Research Personnel

T. S. Angell – Principal Investigator

R. E. Kleinman – Principal Investigator

P. M. van den Berg – Scientific Investigator

Wen Lixin – Graduate Student

5.a. Presentations supported under AFOSR Grant;

1 July 1994 – 30 June 1995.

1. Determination of the Shape of an Unknown Perfectly Conducting Object from Experimental Scattered Field Data, P. M. van den Berg and M. G. Coté and R. E. Kleinman, PIERS '94, July 1994, Noordwijk, Netherlands.

2. A Nonlinearized Iterative Approach of the Eddy Current Characterization of Voids in a Conductive Half Space, R. E. Kleinman, B. Duchêne and D. Lesselier, PIERS '94, July 1994, Noordwijk, Netherlands.

3. On the Location of an Object from Spatially Limited Multifrequency Scattering Data, K. Belkebir and R. E. Kleinman, PIERS '94, July 1994, Noordwijk, Netherlands.

4. A Nonlinearized Iterative Approach of the Acoustic Characterization of Defects Within a Half-Space, B. Duchêne, D. Lesselier and R. E. Kleinman, IEEE Ultrasonics Symposium, November 1994, Cannes, France.

5. Optimal Current Distribution for a Conformal Antenna at High Frequencies, T. S. Angell, R. E. Kleinman and B. Vainberg, URSI National Radio Science Meeting, Jan. 1995, Boulder, CO.

6. Two Dimensional Profile Inversion - The TE Case, L. Wen, R. E. Kleinman and P. M. van den Berg, URSI International Symposium on Electromagnetic Theory, May 1995, St. Petersburg, Russia.

7. Image Reconstruction from Ipswich Data, P. M. van den Berg and R. E. Kleinman, 1995 IEEE AP-S Symposium - USNC/URSI Meeting, June 1995, Newport Beach, CA.

8. Object Reconstruction from Far-Field Data Using Gradient and Gauss-Newton Type Methods, P. Lobel, Ch. Pichot, R. E. Kleinman, L. Blanc-Féraud and M. Barlaud, 1995 IEEE AP-S Symposium - USNC/URSI Meeting, June 1995, Newport Beach, CA.

9. Modified Gradient Profile Inversion Using H-Polarized Waves, P. M. van den Berg, R. E. Kleinman and L. Wen, 1995 IEEE AP-S Symposium - USNC/URSI Meeting, June 1995, Newport Beach, CA.

10. Differential Spatial Iterative Methods for Microwave Inverse Scattering, P. Lobel, Ch. Pichot, R. E. Kleinman, L. Blanc-Féraud and M. Barlaud, 1995 IEEE AP-S Symposium - USNC/URSI Meeting, June 1995, Newport Beach, CA.

11. Gradient Method for Solving Non–Linear Inverse Scattering in Microwave Tomography, P. Lobel, R. E. Kleinman, Ch. Pichot, L. Blanc–Ferand and M. Barlaud, PIERS '95, Seattle, Washington, 1995.

12. Gradient Methods in Inverse Acoustic and Electromagnetic Scattering, Peter van den Berg and Ralph Kleinman, IMA Summer Program in Large Scale Optimization, July 1995 (to appear in proceedings).

13. Inverse Scattering in Acoustics and Electromagnetics, Ralph E. Kleinman, XXVIII National Congress of the Mexican Mathematical Society, Colima, Mexico, October 1995.

14. Error Measures in the Numerical Solution of Integral Equation Formulations of Scattering and Potential Problems, Ralph E. Kleinman, XXVIII National Congress of the Mexican Mathematical Society, Colima, Mexico, October 1995.

5.b. Interactions with other Institutions

Rome Laboratory, Hanscom AFB: "Blind" reconstruction test of the modified gradient inversion algorithm was carried out with experimental data measured at the

Ipswich facility of Hanscom. A special session at the IEEE–AP–S/URSI meeting in Newport Beach, CA in June 1995 on Image Reconstruction from Real Data wherein experimental data, provided on an anonymous FTP server from Hanscom, was organized jointly with Hanscom, Delaware and University of Massachusetts, Lowell. A second such special session was organized at the IEEE–AP–S/URSI meeting in Baltimore, July 1996. The point of contact at Hanscom is R. V. McGahan.

Laboratory for Electromagnetic Research, Delft University of Technology, the Netherlands: All of the original work on the modified gradient approach to inverse problems has been carried out in collaboration with Peter van den Berg.

Institute for Applied Mathematics, University of Erlangen, Nürnberg, Germany: Andreas Kirsch of the Institute has collaborated on the work on resistive boundary conditions and is currently involved in a joint book project on optimization methods in antenna theory. Frank Hettlich is also collaborating on work leading to computational methods for resistive and conductive problems using new characterizations of domain derivatives. A paper is in preparation.

Laboratoire des Signaux et Systêmes, CNRS, Ecole Superìeure d'Electricité, France: Collaboration with D. Lesselier and B. Duchêne is under way on extending inversion methods to reconstructing the shape and location of objects buried in a half space from multifrequency data collected in the complementary half space.

Electronics Laboratory, University of Nice–Sophia Antipolis: A hybrid method combining the modified gradient method with the Newton–Kantorovich approach to profile reconstruction has been developed in collaboration with Christian Pichot.

University of North Carolina at Charlotte: Work on the high frequency asymptotic approach to optimal current determination as well as two dimensional low frequency asymptotics was carried out in collaboration with Boris Vainberg.

University of Patras: Ongoing collaboration with George Dassios is concerned with a monograph on low frequency scattering.

Laboratory of Mechanics and Acoustics, CNRS, France: Extension of the complete family approach to inverse problems to buried objects is under way with Armand Wirgin.

Technical University of Denmark: Work on point collocation was carried out with Søren Christiansen.